# 10.4 Inverse of a Function

**Essential Questions:**

How are a function and its inverse related?

**Learning Goals:**

- I can find inverses of relations.
- I can explore inverses of functions.
- I can find inverses of functions algebraically.
- I can find inverses of nonlinear functions.

An inverse function switches the input and output values.

Inverse functions are functions that "undo" each other.

**I can find inverses of relations**

**Example 1:** Finding Inverses of Relations

1) (-2, 6) (1, 5) (4, 4) (7, 3) (10, 2)

(6,-2) (5,1) (4,4) (3,7) (2,10) (inverse)

2)

| Input | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| Output | 0 | 2 | 4 | 4 | 2 | 0 |

| Input | 0 | 2 | 4 | 4 | 2 | 0 |
|---|---|---|---|---|---|---|
| Output | 0 | 2 | 4 | 6 | 8 | 10 |

You-Try:

1) (-3, -4) (-2, 0) (-1, 4) (0, 8) (1, 12) (2, 16)

(-4,-3) (0,-2) (4,-1) (8,0) (12,1) (16,2)

2)

| Input | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| Output | 4 | 1 | 0 | 1 | 4 |

| Input | 4 | 1 | 0 | 1 | 4 |
|---|---|---|---|---|---|
| Output | -2 | -1 | 0 | 1 | 2 |

**I can explore inverses of functions**

**Example 2:**

1) Let $f(x) = 3x - 5$. Solve $y = f(x)$ for x. Then find the input when the output is 4.

$y = 3x - 5$

$\frac{y+5}{3} = \frac{3x}{3}$

$x = \frac{y+5}{3}$

$x = \frac{4+5}{3}$

$x = \frac{9}{3}$

$\boxed{x = 3}$

The input is 3 when the output is 4

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2) $f(x) = \frac{1}{2}x + 3$

$y = \frac{1}{2}x + 3$

$y - 3 = \frac{1}{2}x$

$\boxed{2(y-3) = x}$

output = 4

$2(4-3) = x$

$2(1) = x$

$\boxed{2 = x}$

3) $f(x) = 4x^2$

$y = 4x^2$

$\frac{y}{4} = x^2$

$\boxed{x = \pm\sqrt{\frac{y}{4}}}$

Output = 4

$x = \pm\sqrt{\frac{4}{4}}$

$x = \pm\sqrt{1}$

$\boxed{x = \pm 1}$

You-Try:

1) Solve $y = f(x)$ for x. Then find the input when the output is 2 for: $f(x) = 2x - 3$

$y = 2x - 3$

$\frac{y+3}{2} = \frac{2x}{2}$

$\frac{y+3}{2} = x$

$\frac{(2+3)}{2} = x$

$\boxed{\frac{5}{2} = x}$

**I can find inverses of functions algebraically**

Writing a Formula for the Input of a Function

Step 1: Let $y = f(x)$ ⇒ Change f(x) to y

Step 2: switch x and y

Step 3: Solve for y

Step 4: Find the input, if asked

Domain and Range are also switched.

**Example 3:** Finding the Inverse of a Linear Function

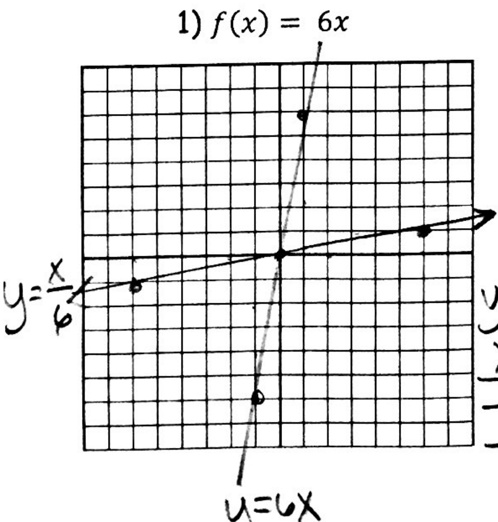
Find the inverse of the following linear functions. Then graph both functions.

1) $f(x) = 6x$

$y = 6x$ (original)

$\frac{x}{6} = \frac{6y}{6}$

$y = \frac{x}{6}$ (inverse)




$y = 6x$

| x | y |
|---|---|
| -2 | -12 |
| -1 | -6 |
| 0 | 0 |
| 1 | 6 |
| 2 | 12 |

$y = \frac{x}{6}$

| x | y |
|---|---|
| -6 | -1 |
| -3 | -.5 |
| 0 | 0 |
| 3 | .5 |
| 6 | 1 |

2) $f(x) = 3x + 4$

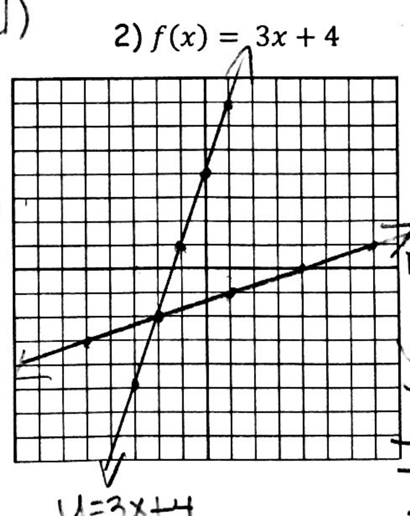
(original)

$y = 3x + 4$

$x = 3y + 4$

$\frac{x-4}{3} = \frac{3y}{3}$

$y = \frac{x-4}{3}$




$y = 3x+4$

| x | y |
|---|---|
| -3 | -5 |
| -2 | -2 |
| -1 | 1 |
| 0 | 4 |
| 1 | 7 |

$y = \frac{x-4}{3}$

| x | y |
|---|---|
| -5 | -3 |
| -2 | -2 |
| 1 | -1 |
| 4 | 0 |
| 7 | 1 |

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You-Try:

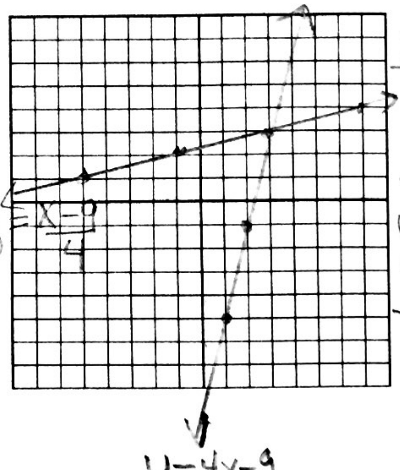
1) $f(x) = 4x - 9$

$y = 4x - 9$ (orig)

$x = 4y - 9$

$\frac{x+9}{4} = \frac{4y}{4}$

$y = \frac{x+9}{4}$



$y = \frac{x-9}{4}$

$y = 4x - 9$

$y = 4x - 9$

| x | y |
|---|---|
| 0 | -9 |
| 1 | -5 |
| 2 | -1 |
| 3 | 3 |

$y = \frac{x-9}{4}$

| x | y |
|---|---|
| -5 | 1 |
| -1 | 2 |
| 3 | 3 |
| 7 | 4 |

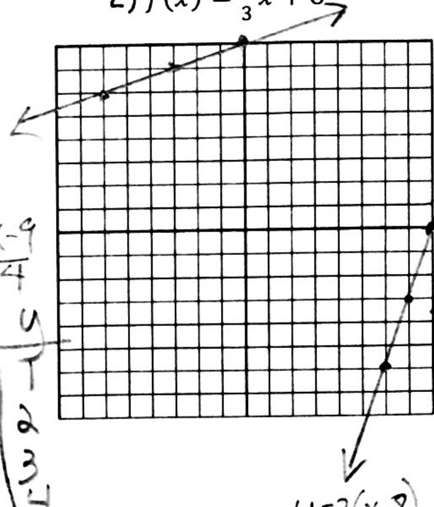
2) $f(x) = \frac{1}{3}x + 8$

$y = \frac{1}{3}x + 8$

$x = \frac{1}{3}y + 8$

$x - 8 = \frac{1}{3}y$

$3(x-8) = y$



$y = \frac{1}{3}x + 8$

| x | y |
|---|---|
| -6 | 6 |
| -3 | 7 |
| 0 | 8 |
| 3 | 9 |

$y = 3(x-8)$

$y = 3(x-8)$

| x | y |
|---|---|
| 6 | -6 |
| 7 | -3 |
| 8 | 0 |
| 9 | 3 |

**I can find inverses of nonlinear functions**

**Example 4:** Finding the Inverse of a Quadratic Function

Find the inverse of the following quadratic functions. Then graph both functions.

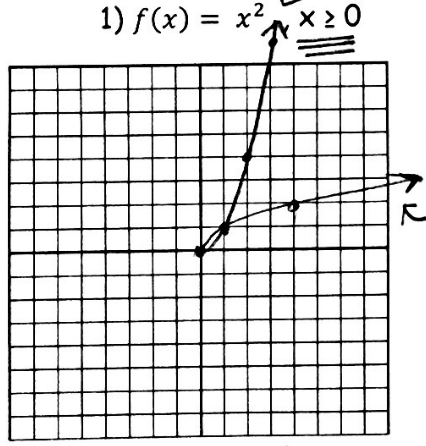
1) $f(x) = x^2, x \geq 0$

$y = x^2$

$\sqrt{x} = \sqrt{y^2}$

$\sqrt{x} = y$

$y = \sqrt{x}$



| x | y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |

$y = x^2 \ (x \geq 0)$

| x | y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

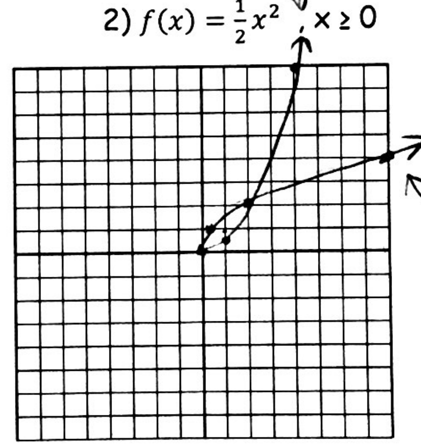
2) $f(x) = \frac{1}{2}x^2, x \geq 0$

$y = \frac{1}{2}x^2$

$x = \frac{1}{2}y^2$

$\sqrt{2x} = \sqrt{y^2}$

$y = \sqrt{2x}$



| x | y |
|---|---|
| 0 | 0 |
| $\frac{1}{2}$ | 1 |
| 2 | 2 |
| 8 | 4 |

$y = \frac{1}{2}x^2$

| x | y |
|---|---|
| 0 | 0 |
| 1 | $\frac{1}{2}$ |
| 2 | 2 |
| 4 | 8 |

# 10.4 Inverse of a Function

You-Try:

1) $f(x) = -x^2 + 10$ , $x \le 0$

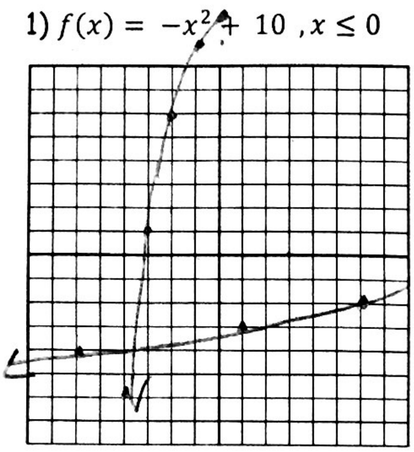
$y = -x^2 + 10$ (org)

$x = -y^2 + 10$

$x - 10 = -y^2$

$10 - x = y^2$

$\sqrt{10 - x} = y$



$y = -x^2 + 10$

| x | y |
|---|---|
| 0 | 10 |
| -1 | 9 |
| -2 | 6 |
| -3 | 1 |

$y = \sqrt{10 - x}$

| x | y |
|---|---|
| 10 | 0 |
| 9 | -1 |
| 6 | -2 |
| 1 | -3 |
| -6 | -4 |

2) $f(x) = 2x^2 + 6, x \ge 0$

$y = 2x^2 + 6$ $(x \ge 0)$

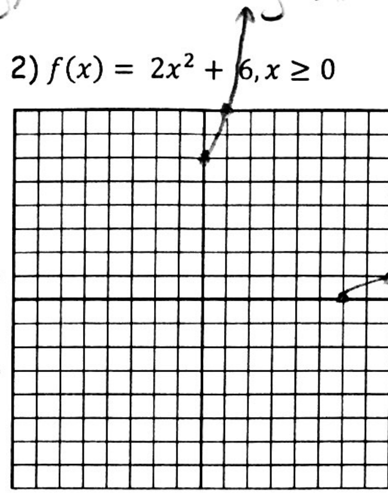
$y = 2x^2 + 6$

$x = 2y^2 + 6$

$\frac{x - 6}{2} = \frac{2y^2}{2}$

$y^2 = \frac{x - 6}{2}$

$y = \sqrt{\frac{x - 6}{2}}$



orig

| x | y |
|---|---|
| 0 | 6 |
| 1 | 8 |
| 2 | 14 |

inverse

| x | y |
|---|---|
| 6 | 0 |
| 8 | 1 |
| 14 | 2 |

**Horizontal Line Test:** The inverse of a function f is also a function if and only if no horizontal line intersects the graph of the original function more than once.

**Example 5:** Finding the Inverse of a Radical Function

Find the inverse of the function. Then graph the function and its inverse to deterime if the inverse is a function.

1) $f(x) = \sqrt{x + 2}$

$x \ge -2, y \ge 0$

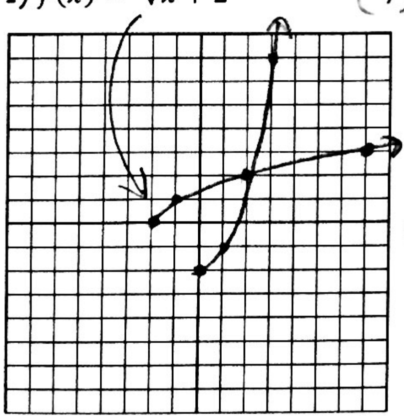
$y = \sqrt{x + 2}$

$(x = \sqrt{y + 2})^2$

$x^2 = y + 2$

$y = x^2 - 2$

$x \ge 0, y \ge -2$



$y = \sqrt{x + 2}$

| x | y |
|---|---|
| -2 | 0 |
| -1 | 1 |
| 2 | 2 |
| 7 | 3 |

$y = x^2 - 2$

$x \ge 0$

| x | y |
|---|---|
| 0 | -2 |
| 1 | -1 |
| 2 | 2 |
| 3 | 7 |

You-Try: $f(x) = \sqrt{2x - 1}$

$x \ge \frac{1}{2}$ $y \ge 0$

$y = \sqrt{2x - 1}$

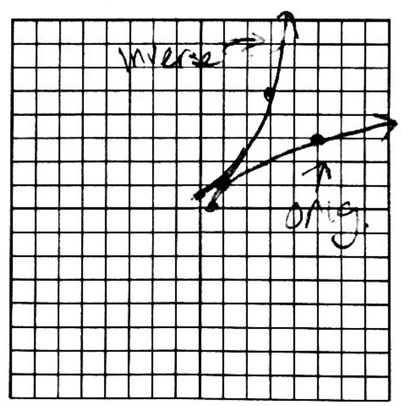
$x = \sqrt{2y - 1}$

$x^2 = 2y - 1$

$x^2 + 1 = 2y$

$\frac{x^2 + 1}{2} = y$

$x \ge 0$ $y \ge \frac{1}{2}$




org

| x | y |
|---|---|
| $\frac{1}{2}$ | 0 |
| 1 | 1 |
| 5 | 3 |

inverse

| x | y |
|---|---|
| 0 | $\frac{1}{2}$ |
| 1 | 1 |
| 3 | 5 |

**Closure:** What I learned today was…..