

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

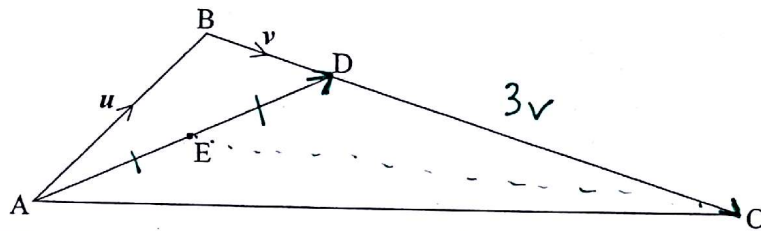
SECTION A

[Maximum mark: 44]

Answer all questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

In the following diagram, $u = \vec{AB}$ and $v = \vec{BD}$.



The midpoint of \vec{AD} is E and $\frac{BD}{DC} = \frac{1}{3}$.

$BD = \frac{1}{3}DC$

Express each of the following vectors in terms of u and v .

(a) \vec{AE}

[3 marks]

(b) \vec{EC}

[4 marks]

a) $\vec{AE} = \frac{1}{2}\vec{AD}$ $\vec{AD} = u + v$
 $= \frac{1}{2}(u + v)$
 $= \frac{1}{2}u + \frac{1}{2}v$

b) $\vec{EC} = \vec{ED} + \vec{DC}$
 $= \vec{AE} + \vec{DC}$
 $= \frac{1}{2}u + \frac{1}{2}v + 3v$
 $= \frac{1}{2}u + \frac{7}{2}v$

2. [Maximum mark: 5]

There are nine books on a shelf. For each book, x is the number of pages, and $\pounds y$ is the selling price. Let r be the correlation coefficient.



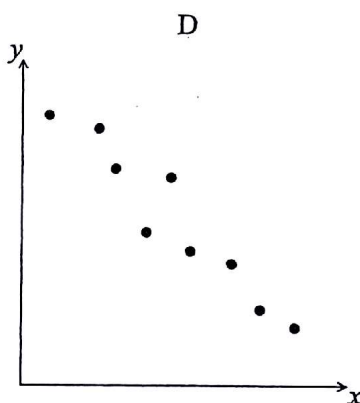
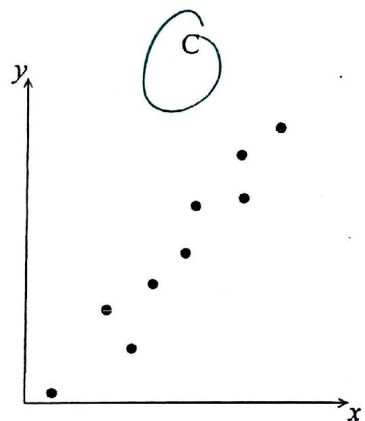
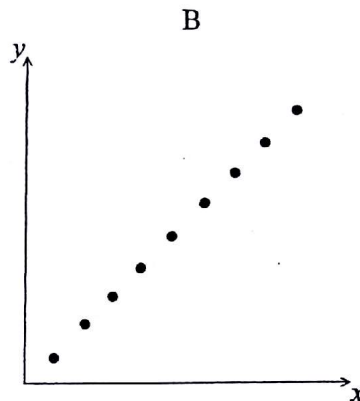
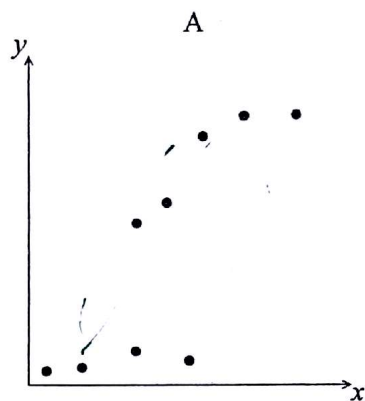
(a) Write down the possible minimum and maximum values of r .

[2 marks]

$$-1 \leq r \leq 1$$

(b) Given that $r = 0.95$, which of the following diagrams best represents the data.

[1 mark]



(c) For the data in diagram D, which two of the following expressions describe the correlation between x and y ?

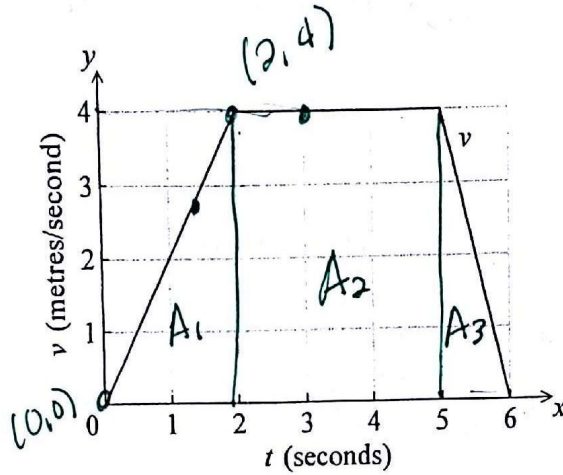
perfect, zero, linear, strong positive, strong negative,
weak positive, weak negative

[2 marks]

(This question continues on the following page)

3. [Maximum mark: 6]

A toy car travels with velocity $v \text{ ms}^{-1}$ for six seconds. This is shown in the graph below.



- (a) Write down the car's velocity at $t=3$. 4 m/s [1 mark]
- (b) Find the car's acceleration at $t=1.5$. [2 marks]
- (c) Find the total distance travelled. [3 marks]

area

b) $\frac{4-0}{2-0} = \frac{4}{2} = 2 \text{ m/s}^2$

c) $A_1 + A_2 + A_3$
 $\frac{1}{2}(2)(4) + 3(4) + \frac{1}{2}(1)(4)$
 $4 + 12 + 2 = 18 \text{ m}$

4. [Maximum mark: 5]

A set of data has a mean of 20 and a standard deviation of 6.

(a) Each value in the data set has 10 added to it. Write down the value of

- (i) the new mean;
- (ii) the new standard deviation.

[2 marks]

(b) Each value in the original data set is multiplied by 10.

- (i) Write down the value of the new mean.
- (ii) Find the value of the new variance.

[3 marks]

a) i) $20 + 10 = 30$
ii) 6

b) i) $20 \cdot 10 = 200$
ii) $6 \cdot 10 = 60$
variance = 60^2
 $= 3600$

Turn over

5. [Maximum mark: 7]

(a) Find $\int \frac{e^x}{1+e^x} dx$.

[3 marks]

(b) Find $\int \sin 3x \cos 3x dx$.

[4 marks]

a) $\int \frac{e^x}{1+e^x} dx$ $u = 1+e^x$
 $du = e^x dx$

$\int \frac{1}{u} du = \ln|u| + C$
 $\ln|1+e^x| + C$
 $\ln(1+e^x) + C$

b) $\int \sin 3x \cos 3x dx$ $u = \sin 3x$
 $du = 3 \cos 3x dx$
 $\frac{1}{3} du = \cos 3x dx$

$\int u \cdot \frac{1}{3} du = \frac{1}{3} \int u du$

$\frac{1}{3} (\frac{1}{2} u^2) = \frac{1}{6} \sin^2 3x + C$

6. [Maximum mark: 7]

The expression $6 \sin x \cos x$ can be expressed in the form $a \sin bx$.

(a) Find the value of a and of b .

[3 marks]

(b) Hence or otherwise, solve the equation $6 \sin x \cos x = \frac{3}{2}$, for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$.

[4 marks]

a) $6 \sin x \cos x = 3 \boxed{2 \sin x \cos x}$
 $3 \boxed{\sin 2x}$
 $3 \sin 2x \quad a=3$
 $\uparrow \quad \uparrow \quad b=2$
 $a \quad b$

b) $6 \sin x \cos x = 3 \sin 2x \quad 3 \sin 2x = \frac{3}{2}$

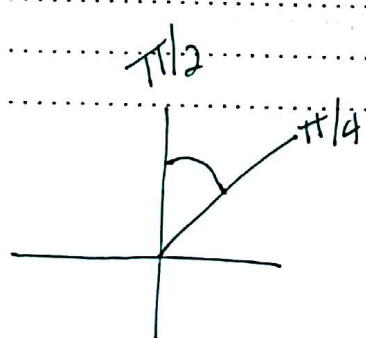
$\sin 2x = \frac{1}{2}$

$\sin^{-1}(\frac{1}{2}) = 2x$

$2x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}$

$x = \frac{5\pi}{12}$



Turn over

7. [Maximum mark: 7]

Given that $f(x) = \frac{1}{x}$, answer the following.

(a) Find the first four derivatives of $f(x)$. [4 marks]

(b) Write an expression for $f^{(n)}(x)$ in terms of x and n . [3 marks]

$$\begin{aligned} a) f(x) &= x^{-1} \\ f'(x) &= -x^{-2} = -\frac{1}{x^2} \\ f''(x) &= 2x^{-3} = \frac{2}{x^3} \\ f'''(x) &= -6x^{-4} = -\frac{6}{x^4} \\ f^{(4)}(x) &= 24x^{-5} = \frac{24}{x^5} \end{aligned}$$

$$b) (-1)^n n! x^{-n-1}$$

Do NOT write solutions on this page. Any working on this page will NOT be marked.

SECTION B

[Maximum mark: 46]

Answer all the questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 15]

Let $f(x) = 3(x+1)^2 - 12$.

$3(x^2 + 2x + 1) - 12$

$3x^2 + 6x + 3 - 12$

$3x^2 + 6x - 9$ ✓

(a) Show that $f(x) = 3x^2 + 6x - 9$.

[2 marks]

(b) For the graph of f

(i) write down the coordinates of the vertex; $(-1, -12)$

(ii) write down the y-intercept; -9 $f(0) = 3(0)^2 + 6(0) - 9 = -9$

(iii) find both x-intercepts. $0 = 3x^2 + 6x - 9$

[7 marks]

(c) Hence sketch the graph of f .

$3(x^2 + 2x - 3)$ $x = -3, 1$

$3(x+3)(x-1)$

[3 marks]

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the following two transformations:

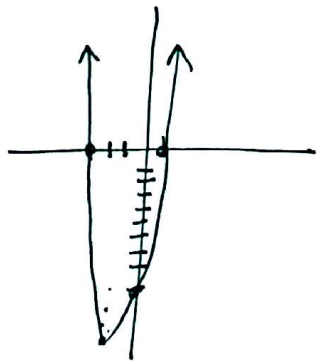
- a stretch of scale factor t in the y -direction,
- followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t .

[3 marks]

$p = -1$
 $q = -12$
 $t = 3$

$f(2x)$



Turn over

Do NOT write solutions on this page. Any working on this page will NOT be marked.

9. [Maximum mark: 14]

Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.

11
C

		Score on second die					
		1	2	3	4	5	6
Score on first die	1	.2	.3	.4	.5	.6	.7
	2	.3	.4	.5	.6	.7	.8
	3	.4	.5	.6	.7	.8	.9
	4	.5	.6	.7	.8	.9	.10
	5	.6	.7	.8	.9	.10	.11
	6	.7	.8	.9	.10	.11	.12

Let X be the sum of the scores on the two dice.

- (a) (i) Find $P(X = 6)$. $\frac{5}{36}$
 - (ii) Find $P(X > 6)$. $\frac{21}{36} = \frac{7}{12}$
 - (iii) Find $P(X = 7 | X > 6)$. $\frac{\frac{6}{36}}{\frac{21}{36}} = \frac{6}{21} = \frac{2}{7}$
- (b) Elena plays a game where she tosses two dice.

[6 marks]

If the sum is 6, she wins 3 points.
 If the sum is greater than 6, she wins 1 point.
 If the sum is less than 6, she loses k points.

Find the value of k for which the game is fair.

[8 marks]

$$36 \left(\frac{5}{36}(3) + \frac{21}{36}(1) - \frac{10}{36}(k) \right) = 0 \cdot 36$$

$$15 + 21 - 10k = 0$$

$$36 - 10k = 0$$

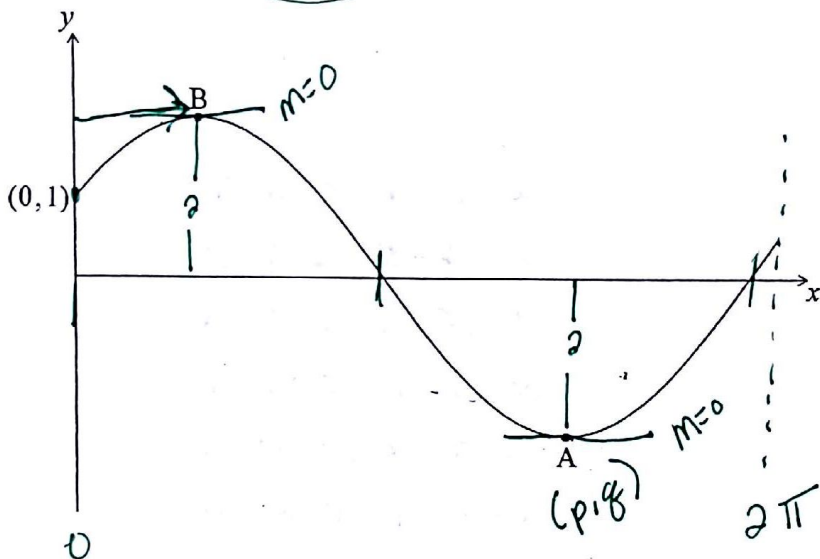
$$36 = 10k$$

$$3.6 = k$$

Do NOT write solutions on this page. Any working on this page will NOT be marked.

10. [Maximum mark: 16]

Let $f(x) = \cos x + \sqrt{3} \sin x$, $0 \leq x \leq 2\pi$. The following diagram shows the graph of f .



The y -intercept is at $(0, 1)$, there is a minimum point at $A (p, q)$ and a maximum point at B .

- (a) Find $f'(x)$. $f'(x) = -\sin x + \sqrt{3} \cos x$ [2 marks]
- (b) Hence
- (i) show that $q = -2$; $\frac{\sin x}{\cos x} = \sqrt{3} = \tan x$ [10 marks]
- (ii) verify that A is a minimum point.

- (c) Find the maximum value of $f(x)$. $f(\frac{\pi}{3}) = \cos \frac{\pi}{3} + \sqrt{3} \sin \frac{\pi}{3}$ [3 marks]

The function $f(x)$ can be written in the form $r \cos(x-a)$. $\frac{1}{2} + \sqrt{3} \frac{\sqrt{3}}{2} = \frac{1}{2} + \frac{3}{2} = 2$

- (d) Write down the value of r and of a . [2 marks]

$r = 2$
 $a = \frac{\pi}{3}$

Trig-graphs
Trig-expressions
Vectors
Derivatives

Integrals

Turn over