Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A *[Maximum Mark: 47]*

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

u_1 S_{20}

In an arithmetic series, the first term is -7 and the sum of the first 20 terms is 620.

(a) Find the common difference. *[3 marks]*

(b) Find the value of the 78^{th} term. *[2 marks]*

$S_n = \frac{n}{2}(2u_1 + (n-1)d)$

$S_{20} = \frac{20}{2}(2(-7) + (20-1)d) = 620$

$d = 4$

$u_n = u_1 + (n-1)d$

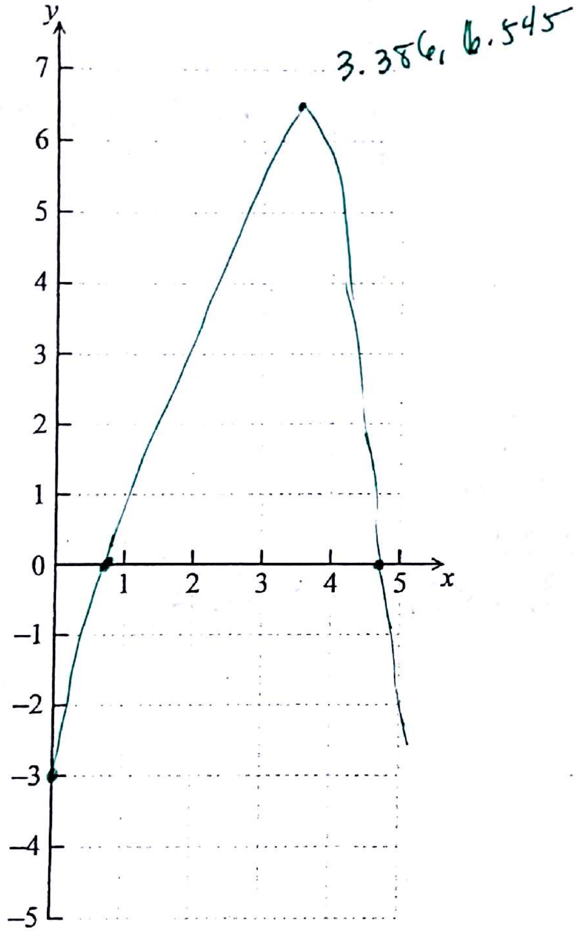
b) $u_{78} = -7 + (78-1) \cdot 4 = 301$

2. *[Maximum mark: 7]*

Let $f(x) = 4x - e^{x-2} - 3$, for $0 \le x \le 5$.

(a) Find the x-intercepts of the graph of f. using calc (0.827, 0) (4.78, 0) *[3 marks]*

(b) On the grid below, sketch the graph of f. *[3 marks]*



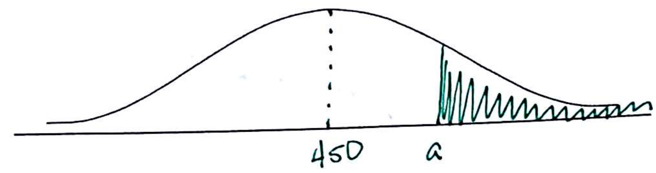
(c) Write down the gradient of the graph of f at $x = 3$. *[1 mark]*

using calc gradient = 1.28
calc → $\frac{dy}{dx}$

3. *[Maximum mark: 6]*

A random variable X is distributed normally with mean 450. It is known that $\mathrm{P}(X > a) = 0.27$.

(a) Represent all this information on the following diagram. *[3 marks]*



(b) Given that the standard deviation is 20, find a. Give your answer correct to the nearest whole number. *[3 marks]*

$X \sim N(450, 20^2)$

$P(X < a) = 1 - 0.27$

$P(X < a) = 0.73$

$\text{invNorm}(.73, 450, 20) = \boxed{462}$

4. *[Maximum mark: 7]*

Consider the lines L_1, L_2, L_3, and L_4, with respective equations

$$L_1:\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}, \quad L_2:\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + p\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix},$$

$$L_3:\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} -1 \\ 2 \\ -a \end{pmatrix}, \quad L_4:\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = q\begin{pmatrix} -6 \\ 4 \\ -2 \end{pmatrix}$$

(a) Write down the line which is parallel to L_4. *[1 mark]*

(b) Write down the position vector of the point of intersection of L_1 and L_2. *[1 mark]*

(c) Given that L_1 is perpendicular to L_3, find the value of a. *[5 marks]*

a) L_1 $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \cdot -2 = \begin{pmatrix} -6 \\ 4 \\ -2 \end{pmatrix}$ ✓

b) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

c) $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ -a \end{pmatrix} = 0$

$-3-4-a=0$

$\boxed{a=-7}$

5. *[Maximum mark: 7]*

The probability of obtaining heads on a biased coin is 0.4 . The coin is tossed 600 times.

(a) (i) Write down the mean number of heads.

(ii) Find the standard deviation of the number of heads. *[4 marks]*

(b) Find the probability that the number of heads obtained is less than one standard deviation away from the mean. *[3 marks]*

a) i) $.4(600) = 240$

ii) $\sqrt{np(1-p)} = \cancel{\sqrt{2486}}$ $\sqrt{600(.4)(.6)} = 12$

b) $P(228 < x < 252) = P(x \leq 251) - P(x \leq 228)$

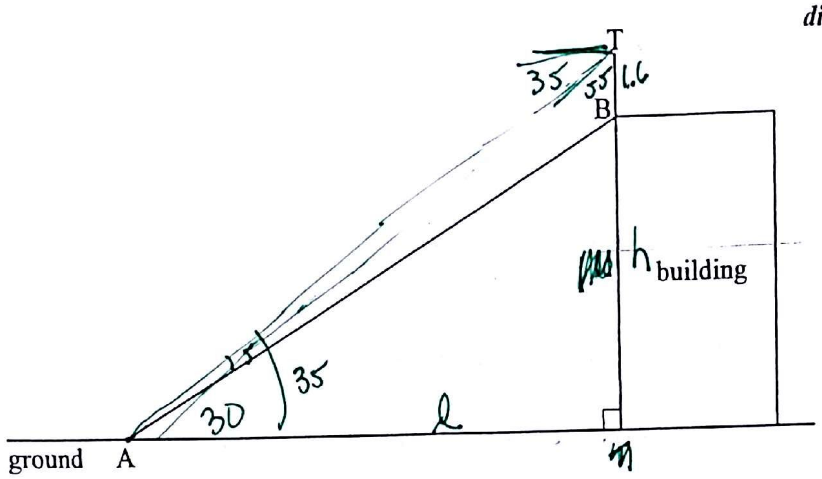
$= .662$

☆ Note- not normal, so must be binomial ☆

6. *[Maximum mark: 7]*

The following diagram shows a pole BT 1.6 m tall on the roof of a vertical building. The angle of depression from T to a point A on the horizontal ground is 35°. The angle of elevation of the top of the building from A is 30°.

diagram not to scale



Find the height of the building.

$\tan 30 = \frac{h}{x}$ $\tan 35 = \frac{h+1.6}{x}$ $\sin 30 = \frac{h}{l}$

$x = \frac{h}{\tan 30}$ $x = \frac{h+1.6}{\tan 35}$

$\cancel{\frac{\sin 30}{1.6+h}}$

$\frac{h}{\tan 30} = \frac{h+1.6}{\tan 35}$

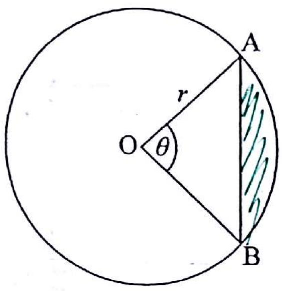
$\tan 35 h = \tan 30 h + \tan 30(1.6)$

$.123h = .924$

$\boxed{h = 7.52\text{m}}$

7. *[Maximum mark: 8]*

A circle centre O and radius r is shown below. The chord [AB] divides the area of the circle into two parts. Angle AOB is θ.



(a) Find an expression for the area of the shaded region. *[3 marks]*

(b) The chord [AB] divides the area of the circle in the ratio 1:7. Find the value of θ. *[5 marks]*

a) $\frac{1}{2}\theta r^2 - \frac{1}{2} r \cdot r \cdot \sin\theta = \boxed{\frac{1}{2} r^2(\theta - \sin\theta)}$

b) ~~triangle~~ shaded is 1/8 of circle

$\frac{1}{2}\cancel{r^2}(\theta - \sin\theta) = \frac{1}{8}\pi \cancel{r^2}$

$\theta - \sin\theta = \frac{\pi}{4}$

on calc $y_1 = x - \sin x$

$y_2 = \frac{\pi}{4}$ $\quad x = 1.77$ radians

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B	*[Maximum Mark: 43]*

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

Each day, a clothing factory recorded the number (x) of boxes it produces and the total production cost (y) dollars. The results for nine days are shown in the following table.

x	26	44	65	43	50	31	68	46	57
y	400	582	784	625	699	448	870	537	724

(a) Write down the equation of the regression line of y on x. *[2 marks]*

on calc $y = 10.7x + 121$

Use your regression line as a model to answer the following.

(b) Interpret the meaning of

(i) the gradient; the cost of production / box

(ii) the y-intercept. the cost for materials, etc. *[2 marks]*

(c) Estimate the cost of producing 60 boxes. *[2 marks]*

$y = 10.7(60) + 121 = \$760$

(d) The factory sells the boxes for \$19.99 each. Find the least number of boxes that the factory should produce in one day in order to make a profit. *[3 marks]*

$19.99x > 10.7x + 121$ $x > 12.94$ $x = 13$ etc...

(e) Comment on the appropriateness of using your model to

(i) estimate the cost of producing 5000 boxes;

extrapolation, not reliable

(ii) estimate the number of boxes produced when the total production cost is \$540. *[4 marks]*

interpolation, reliable

Do NOT write solutions on this page. Any working on this page will NOT be marked.

9. *[Maximum mark: 16]*

Let $h(x)=\dfrac{2x-1}{x+1},\ x\neq -1$.

$x=\dfrac{2y-1}{y+1}$

$xy+x=2y-1$

$\cancel{xdy}$

$xy-2y=-x-1$

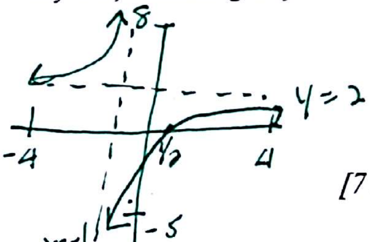
$y(x-2)=-x-1$

$y=\dfrac{-x-1}{x-2}$

$\boxed{y=\dfrac{x+1}{2-x}}$

(a) Find $h^{-1}(x)$. *[4 marks]*

(b) (i) Sketch the graph of h for $-4\leq x\leq 4$ and $-5\leq y\leq 8$, including any asymptotes.



(ii) Write down the equations of the asymptotes.

(iii) Write down the x-intercept of the graph of h. *[7 marks]*

(c) Let R be the region in the first quadrant enclosed by the graph of h, the x-axis and the line $x=3$.

(i) Find the area of R.

$\int_{1/2}^{3}\frac{2x-1}{x+1}\,dx \rightarrow 2.06$

(ii) Write down an expression for the volume obtained when R is revolved through 360° about the x-axis. *[5 marks]*

$\pi\int_{1/2}^{3}\left(\frac{2x-1}{x+1}\right)^2 dx$

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 14]*

A rock falls off the top of a cliff. Let h be its height above ground in metres, after t seconds.

The table below gives values of h and t.

t (seconds)	1	2	3	4	5
h (metres)	105	98	84	60	26

(a) Jane thinks that the function $f(t) = -0.25t^3 - 2.32t^2 + 1.93t + 106$ is a suitable model for the data. Use Jane's model to

(i) write down the height of the cliff; 106

(ii) find the height of the rock after 4.5 seconds;

$f(4.5) = 44.9$

(iii) find after how many seconds the height of the rock is 30 m. use calc to find intersection *[5 marks]*

$30 = f(t) \rightarrow t = 4.91s$

(b) Kevin thinks that the function $g(t) = -5.2t^2 + 9.5t + 100$ is a better model for the data. Use Kevin's model to find when the rock hits the ground. *[3 marks]*

$0 = g(t)$ on calc, $t = 5.39s$

(c) (i) On graph paper, using a scale of 1 cm to 1 second, and 1 cm to 10 m, plot the data given in the table.

(ii) By comparing the graphs of f and g with the data in the table, explain which function is a better model for the height of the falling rock. *[6 marks]*

look @ graphs - Kevin's goes up initially so Jane's better

-Jane's passes more closely